501. Write an equation, using constants m and c. Then differentiate both sides of it.

502. "Unit length" is length 1. The shaded region is a square. Find its edge lengths using Pythagoras.

503. Interior angles of a quadrilateral add to 2π radians.

504. Substitute for b and then for c. You don't need to use constants of proportionality: it's easier leaving the relationships as proportionality statements.

505. Solve simultaneously, and use Δ.

506. Use the factor theorem.

507. In order for RR to show, the two cards with red faces must be drawn. The probability of this is $\frac{1}{3}$. Consider then the probability that the RG card is laid the right way up.

508. Draw a force diagram for the central section. You can calculate its mass by scaling the total mass by a length scale factor.

509. The notation $\mathrm{f}(x)\,\big|_{x=b}$ is another way of writing $\mathrm{f}(b)$. The bar can be translated as "evaluated at". So, for example,

$$x^2\,\Big|_{x=2} = 4.$$

Consider that $-(p-q) \equiv q-p$.

510. (a) Multiply out the LHS and simplify.
(b) Consider the fact that $(2k^2+2k) \in \mathbb{Z}$.

511. One clock hour or five clock minutes subtends an angle $\frac{2\pi}{12} = \frac{\pi}{6}$ radians.

512. (a) The largest angle is opposite the largest length.
(b) Use $A_\triangle = \frac{1}{2}ab\sin C$.
(c) Use $A_\triangle = \frac{1}{2}\text{base} \times \text{height}$.

513. Each definition should have the word "negligible" in it. The question here is what physical quantity is to be neglected.

514. Find all four values of 2θ in $[0, 4\pi)$.

515. (a) In this non-standard differentiation from first principles, the gradient triangle is set up to the left of the point (x, x^2).
(b) δx is often called h.
(c) Expand and simplify, then cancel a factor of δx. Then take the limit, i.e. let δx go all the way to zero.

516. Use the area formula

$$A_\triangle = \tfrac{1}{2}ab\sin C.$$

Use the first Pythagorean trig identity to convert $\sin C$ into $\cos C$. Then use the cosine rule. You should get two answers, one of which is an integer.

517. One is true, the other false.

518. Getting cards of the same suit is more probable with replacement.

519. Use the fact that, for a circle, tangent and radius are perpendicular.

520. Consider constant velocity as zero acceleration.

521. Simplify the middle term first, writing it as x^k. The equation is a quadratic in $x^{\frac{5}{2}}$.

522. Add, multiply out and simplify.

523. Find the coordinates of the vertices (turning points) of the parabolae. The centre of rotation must be the midpoint of these.

524. Use the formula $\theta = \pi - \frac{2\pi}{n}$.

525. Consider parity, i.e. evenness/oddness.

526. Consider the fact that definite integrals calculate *signed* area, i.e. area with a sign attached, not simply area.

527. An implies arrow means "If ... then ...".

528. The quantity $x^4 + y^4$ is zero at O, and increases with distance from O. So, test it at $(3, 4)$ and compare to 400.

529. Label the sides of the rectangles x, y, and set up and solve simultaneous equations.

530. (a) Include R_1, R_2 and the weight W.
(b) The result you want is a relationship between R_1 and R_2, so you only need one equation here. Take moments around the centre of the beam.

531. Use 3D Pythagoras.

532. (a) Differentiate and sub $x = p$.
(b) Substitute the point (p, p^2) into $y = 2px + c$ and simplify.

533. Consider the symmetry of the kite.

534. There is no need to split up the solution into a three, then a two, then a one. All coin tosses may be assumed to be independent, so, in total, we have a binomial distribution of six coin tosses.

535. One of the integrals calculates the area between $y = x^3$ and the x axis, the other the area between $y = x^3$ and the y axis. Combined, the regions form a rectangle with vertices at $(0, 0)$ and (k, k^3).

536. Show that the number must have a factor of 2 and a factor of 5.

537. Consider the direction of the implication.

538. Differentiate and use the negative reciprocal: the normal is perpendicular to the tangent.

539. Form an equation in n, and solve.

540. Sketch the graph

$$y = (x - 1)(x - 2)(x - 3).$$

This is a positive cubic with distinct x intercepts at $x = 1, 2, 3$. Consider the x values for which the y value is less than or equal to zero.

541. Sketch the triangle, and consider perpendicular heights in $A_\triangle = \frac{1}{2}\text{base} \times \text{height}$.

542. Consider the roots of the numerator.

543. Find and simplify an expression for the common ratio

$$\frac{w_{n+1}}{w_n}$$

of the new sequence. Use the standard nth term formula $u_n = ar^{n-1}$ for a GP.

544. (a) Substitute for y.
(b) The integrand is $2x + 1 - x^2$: this represents a height/distance in the y direction.
(c) Carry out the definite integral, simplifying the surds carefully.

545. The "term independent of x" is another way of saying the constant term. In this expansion, the constant term is the middle term.

546. You could sketch the lines/curves. However, it is easier to give reasons algebraically here. Consider the equation for intersections in each case.

547. If an n-gon and an $(n + 1)$-gon have the same perimeter, then the $(n + 1)$-gon has larger area.

548. Multiply out and simplify.

549. (a) Sketch the boundary equation $x + y = 2$ first, as a dashed line (non-inclusive).
(b) Consider the point $(1, 1)$.
(c) Consider the fact that x and y are each raised to an even power.

550. Factorise as a quadratic in x^2. The quadratic has two roots and the biquadratic has four.

551. Two are normal and two are not. Use the fact that a normal distribution is symmetrical around the mean.

552. Consider $x = 30°$ and $y = 150°$.

553. Put everything on one side, equating to zero. Then complete the square for x and for y.

554. The numerators form a difference of two squares.

555. (a) Set up $a^2 + b^2$. Multiply out and simplify to show that $a^2 + b^2 = c^2$.
(b) Each triple generates such a triangle.

556. Find the nth term formula; set it to 1048576 and solve for n. Then use the sum of a GP, which is

$$S_n = \frac{a(1 - r^n)}{1 - r}.$$

557. (a) Multiply the top out and split the fraction up, before differentiating. Alternatively, use the quotient and chain rules.
(b) Set the derivative to zero and solve. Your equation will contain negative indices: to deal with x^{-k}, multiply both sides by x^k.

558. Consider the sum of the interior angles.

559. Form and manipulate an equation in a, b, c.

560. (a) Carry out the definite integral.
(b) Interpret the fact that the total signed area between the curve and the x axis is 0.

561. A monic parabola $y = \mathrm{f}(x)$ has leading coefficient 1. Use the factor theorem.

562. Its easiest to factorise the LHS directly. If you can't spot the factorisation straight away, use the quadratic formula to find roots, and then use the factor theorem. Then compare coefficients.

563. Consider two separate translations, one in the x direction and one in the y direction.

564. Remember that $\mathrm{f}(a)$ is a numerical value, so these statements can be thought of as

   (a) $y = 0 \iff y^2 = 0$,

   (b) $y = 1 \iff y^2 = 1$.

565. Draw a force diagram for the fridge. By NIII, the reaction acting upwards on the fridge has the same magnitude as the reaction acting downwards on the lift.

566. Set up $\triangle ABC$, and drop a perpendicular from B to the side AC. Calculate the length h of this using trigonometry. Then use $A_\triangle = \frac{1}{2}bh$.

567. You can ignore the input transformations; they do not affect the ranges here.

568. (a) Three points A, B, C are collinear if and only if $\overrightarrow{AB} = k\overrightarrow{BC}$ for some $k \in \mathbb{R}$.

   (b) Draw a clear sketch. Show that the line from $(20, 0)$ to $(0, 10)$ and the line from $(0, 0)$ to $(4, 8)$ are perpendicular.

569. Evaluate the LHS and RHS separately.

570. Consider zero gravity in a spacecraft.

571. Sketch the regions on (x, y) axes.

572. Since the line segments are chords, their endpoints, which have $s \in \{0, 1\}$ and $t \in \{0, 1\}$, must lie on the circle. Substitute these values in.

573. Set up the iteration algebraically as

$$A_{n+1} = 3 + 2A_n.$$

   Subtract A_n from both sides of this to get an expression for the term-to-term difference; divide both sides by A_n to get an expression for the term-to-term ratio. Show that neither is constant.

574. Triangle AXC is congruent to triangle ABC.

575. Solve to find intersections, and determine whether the resulting equation has a double root.

576. (a) The stretch affects the period.

   (b) These are output transformations: the period relates to the input of a function.

   (c) Consider the lowest common multiple.

577. Find the intersections first. Then set up a single definite integral, considering which of the line and curve has the greater y values.

578. List the eight outcomes of the possibility space, and group them as S and S'.

579. Before sketching, take the cube root.

580. (a) Set up NII horizontally and vertically.

   (b) Square the equations from (a) and add them, using Pythagoras to simplify and solve for R. Then, either sub this value back in, or divide the equations and use $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$.

581. Sketch the scenario.

582. Take out a factor of $(x - 1)$.

583. To answer all parts, you could consider the graph $y = 4x^2 + 12x + 11$. Find the vertex and sketch this, then reflect in the line $y = x$.

584. To translate by $5\mathbf{i}$, add 5 to the value of the x coordinate. Alternatively, replace x by $x - 4$.

585. Factorise numerator and denominator.

586. Differentiate both sides of $C = 2\pi r$.

587. (a) Consider the fact that the factors $(x - p)$ and $(x - q)$ are both squared.

   (b) This is a positive quartic which is tangent to the x axis at $x = p$ and $x = q$.

   (c) Consider the symmetry of the curve.

588. Add radii from the centre to two relevant points, forming a right-angled triangle. Then you can use Pythagoras/trigonometry.

589. Use 3D Pythagoras. A “unit vector” is a vector with unit length, i.e. length 1.

590. Multiply both sides by $x(1 - x)(1 + x)$. Then you can compare the constant terms.

591. Use $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$.

592. Multiply out, differentiate, then factorise.

593. Consider the long-term behaviour of an AP.

594. Write down the average value of the function on the domain $[0, 1]$. Use this to work out the required value of $\mathrm{f}(0)$, which will allow you to sketch.

595. Each triangular sector of the octagon is an isosceles triangle. Split one into two, and you have right-angled triangles containing the angle $\frac{\pi}{8}$ radians.

596. Express the square numbers algebraically as a^2 and b^2, where $a, b \in \mathbb{Z}$.

597. Express the factors of the LHS over base 2.

598. (a) A "monic" quadratic has leading coefficient 1. The info at $x = 2$ gives you the vertex.
 (b) Reflect your answer to (a) in $y = x$.

599. To connect vectors tip-to-tail in this fashion is to add them. Explain why the sum must be zero.

600. (a) $45°$ is $\frac{\pi}{4}$ radians.
 (b) One revolution is 2π radians.

——— END OF 6TH HUNDRED ———